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Calculation of travelling chimera speeds for dynamical systems with ring topologies

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***Abstract*—Travelling chimera states are a dynamical regime in homogeneous networks where coherent and incoherent domains coexist and the latter moves across the network with time. For such travelling chimeras we can define its speed as a number of elements by which an incoherent domain is shifted per unit time. In this paper, we propose a new approach to calculate the speed of such traveling chimeras. We validate our method by computing the travelling chimera speed in a ring of type-II Morris-Lecar neurons with asymmetrical nonlocal inhibitory connectivity. The main advantage of our approach is that all computations of the speed can be done automatically, opening new opportunities for large-scale scanning and analysis of parametric regions in dynamic systems.**

***Index Terms*—travelling chimera, spiking neural networks, travelling chimera speed, synchronization**

I. Introduction

Synchronization is an important property of networks of intercoupled active elements. Suppose that we observe a state of a homogeneous network, where one part of the elements is synchronized, while another forms an incoherent domain. In that case, this dynamic regime is called a chimera state. Chimera states were initially found in a ring of coupled phase oscillators [1]. Subsequently these states were identified in coupled systems of elements of different nature [2–5], including the spiking neural networks with mostly ring connectivity topologies [6–12]).

When the incoherent domain moves across the network without any external control, the network exhibits a travelling chimera. Travelling chimeras are observed for different systems of phase oscillators [13, 14], mechanical systems [15], electromagnetic oscillators [16], systems with hierarchical connectivity [17] and neural networks [18, 19].

One of the simplest methods used to identify chimera states is by visualizing activity snapshots and the frequency distribution. An activity snapshot shows the distribution of values of an oscillating variable across the network at a fixed time moment. The frequency distribution is defined as the time-averaged spike rate for each element in the network. Examining it we can easily find the coherent and the incoherent domains, their number, size, and variability depending on the control parameter. Unfortunately, one cannot calculate the frequency distribution directly for travelling chimeras: the fact that the incoherent domain moves smooths the resulting frequency across the network. On the other hand, if we determine the speed of the moving incoherent domain, we can switch to the traveling coordinates along the moving domain. Moving to this travelling coordinate system gives a standard time series according to which we can compute and plot the frequency diagrams.

In order to find the speed of the travelling chimera, one can use the definition based on the Fourier spectra that have been suggested in [17] and shown to work well for neuronal networks (see, for example, [18]). In short, this method is based on the following idea; Gvien that we have an incoherent domain moving along the ring, we can track the movement of the maximum value of the oscillating variable. The speed of this movement will be the speed of the chimera. So, we have to consider the elements oscillating with the maximum frequency for each fixed value of time. The indexes of these elements form the time series. The second peak of Fourier spectrum of this time series divided by the number of incoherent domains represents the chimera speed.

This algorithm is not without drawbacks (Despite its ability to calculate a travelling chimera's speed accurately). First, it is tailored to the ring topology since it assumes the periodic motion of an incoherent domain. Second, its accuracy depends significantly on the length of the time series, which can play a crucial role in dynamics simulations of large systems. Third, the search for the second frequency peak in the Fourier transform is not always trivial due to noise. Fourth, this method assumes that we know in advance the number of incoherent clusters, which in the general case is also not a trivial task.

The above disadvantages have a significant impact when calculating the speeds of travelling chimeras for biologically relevant spike neural networks. Biological relevance implies a large number of elements in the network and a topology that is more complex than a simple ring. Notably, the third and fourth drawbacks make automatic calculations of chimera speeds nearly impossible. In this paper, we propose an alternative approach for calculating the speed of a travelling chimera that is based on the maximization of the largest coherent domain size. This new approach is free from the disadvantages listed above, is easy to implement, and also works automatically for a wide range of travelling chimeras.

II. Network description

While our new method works automatically for complex connectivity topologies, as a paradigmatic example we first explain it for a simple ring network. To validate the new approach, we considered a system that demonstrate a travelling chimera regime – a ring network of type II-excitable Morris-Lecar neurons [20] with the unidirectional nonlocal inhibitory connectivity:

$$\begin{cases} C\dot{V}_i &= I_{app} - g_{Ca} m_i^{inf}(V_i - E_{Ca}) - \\ & -g_K w_i (V_i - E_K) - g_L(V_i - E_L) + I_i^{syn}, \\ \dot{w}_i &= \phi(w_i^{inf} - w_i)\cosh\frac{V_i - V_3}{2V_4} \\ w_i^{inf} &= 0.5\left(1 + \tanh\frac{V_i - V_3}{V_4}\right) \\ m_i^{inf} &= 0.5\left(1 + \tanh\frac{V_i - V_1}{V_2}\right) \end{cases} \quad (1)$$

where $i \in 1..N$, N is number of neurons. V_i is a membrane potential of i-th neuron. w_i and m_i are the fraction of open K^+ and Ca^{2+} channels, respectively. E_K, E_{Ca}, E_L are the reversal potentials for potassium, calcium and leak channels, respectively. g_K, g_{Ca}, g_L are corresponding conductances. I_{app} is an applied current. C is the membrane capacity. The neurons are characterized by type II excitability.

I_i^{syn} is a synaptic current supplied to i-th neuron. The inhibitory $GABA_A$ synapses are given by first order kinetics [21]:

$$\begin{cases} I_i^{syn} &= \frac{g_{syn}}{N}\sum_{j=i+1}^{i+R} x_j(V_R - V_i) \\ \dot{x}_j &= \alpha\frac{1}{1+\exp(-\frac{V_j - V_{syn}}{K_p})}(1 - x_j) - \beta x_j \end{cases} \quad (2)$$

Here V_R is the reversal potential, V_{syn} is the threshold, K_p is the synaptic activation, g_{syn} is the synaptic strength, $r = \frac{R}{N}$ is the connectivity parameter, R is the number of connections, x_j is the fraction of effective neurotransmitter resources released by presynaptic terminal into the synaptic cleft in the active state of the synapse. The topology of the network is a ring. The fixed parameters are presented in Table I.

Table I
The fixed neuronal network parameters

$g_K = 8$ mS/cm^2	$E_K = -80$ mV	$V_1 = -1.2$ mV
$g_{Ca} = 4.4$ mS/cm^2	$E_{Ca} = 120$ mV	$V_2 = 18$ mV
$g_L = 2$ mS/cm^2	$E_L = -60$ mV	$V_3 = 2$ mV
$\phi = 1/25$	$C = 20$ μF/cm^2	$V_4 = 30$ mV
$\alpha = 1.1$	$K_p = 5$	$V_{syn} = 2$ mV
$\beta = 0.19$	$N = 500$	$V_R = -60$ mV

Numerical integration was performed (1-2) using the fixed step Euler method (100 μs). Simulation time was 30 s.

III. Chimera speed calculation

A. Frequency distribution

A convenient and simple way to analyze a chimera state is to construct a frequency diagram. It represents the distribution of frequencies for each network element (in our case it is a neurons), averaged over a fixed time interval:

$$f[V(i,t)] = \frac{1}{T_{total}}\left|\left\{t : V(i,t) = V_{thr} \ \& \ \dot{V}(i,t) > 0\right\}\right|, \quad (3)$$

where $V(i,t)$ is the membrane potential of the i-th neuron, $\dot{V}(i,t)$ is its time derivative, V_{thr} is the value of the threshold after which the spike happens, and T_{total} is the duration of the time series. If we plot this distribution, we can visually evaluate the number of incoherent clusters, their size, presence of neurons that demonstrate subthreshold oscillations.

For travelling chimera states one cannot plot frequency diagrams directly, since the chimera moves across the ring. Given that the speed of the chimera v_{chi}, defined as the number of network elements in the chimera that move per unit of time, is known in advance, we can move to travelling chimera coordinates: $V(i,t) \rightarrow V(i + v_{chi}t, t)$. In these travelling coordinates we de facto freeze the movement of the incoherent cluster and can distinguish clearly the coherent and incoherent groups of neurons. Figure 1 shows a comparison of the raster plots and the frequency distribution before and after moving to the travelling chimera coordinates.

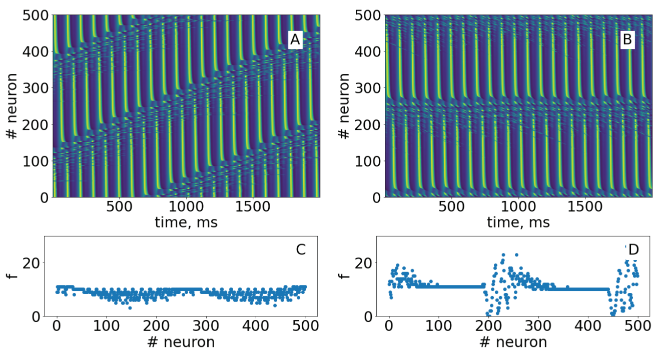

Figure 1. Initial (A) and traveling wave coordinates (B) raster plots in the case of travelling chimera state and corresponding frequency diagrams (C) and (D).

B. Main idea for the chimera speed calculation

We proceed to develop our approach to calculate the chimera speeds that is devoid of the disadvantages associated with the older method (listed above). The most important required property for the new approach is that it must to be automatic.

We propose to develop a heuristic functional $H[v]$, which will measure the quality of the chimera localization for a given speed. Using this functional, we can use it to compute the values of speed v by maximizing the functional directly in a reasonable range of speeds from 0 to v_{max} with a fixed step:

$$v_{chi} = \arg\max_v H[v]. \quad (4)$$

Table II

CALCULATION RESULTS OF THE SPEEDS FOR DIFFERENT FORMS OF THE HEURISTIC FUNCTIONAL AND THE FOURIER-METHOD WERE COMPARED WITH THE EMPIRICAL GROUND TRUTH RESULT. UNSUCCESSFUL EXPERIMENTS ARE MARKED IN RED, WHERE THE OBTAINED SPEED WAS SIGNIFICANTLY DIFFERENT FROM THE GROUND TRUTH. THE TEST NUMBERS CORRESPOND TO THE NUMBERS OF THE CHIMERIC REGIMES SHOWN IN THE SUPPLEMENTARY MATERIALS

	test #1	test #2	test #3	test #4	test #5	test #6	test #7	test #8	test #9	test #10
$H(N_{coh}[v])$	0.0160	0.0803	0	0.0006	0.0204	0.0095	0.0194	0.0085	0	0.0014
$H(N_{inh}[v])$	0.0165	0.0812	0.0420	0	0.0202	0.0095	0	0.0085	0	0.0011
$H(D[v])$	0.0165	0.0803	0.0420	0.0895	0.0723	0.0837	0.0720	0.0590	0.0749	0.0897
$H(N_{coh}[v], N_{inh}[v])$	0.0164	0.0805	0.0420	0	0.0202	0.0095	0.0194	0.0085	0	0.0012
Fourier spectrum method	0.0175	0.0800	0.0417	0.0038	0.05	0.01	0.0512	0.0563	0.0750	0.0275
ground truth	**0.0165**	**0.0805**	**0.0420**	**0**	**0.0202**	**0.0097**	**0.0194**	**0.0085**	**0**	**0.0012**

Next, we consider the possible choices of $H[v]$.

C. Heuristic functional form

Comparing the Fig. 1A and Fig. 1B, we can try to formalize a characteristic that quantifies that the chimera is stationary in the new coordinates and select the heuristic functional.

a) Size of the largest coherent group: For example, based on the reasoning presented above, $H(v)$ can be equal to the size of the largest group of neurons with identical frequency in the travelling chimera coordinates $V(i + vt, t)$:

$$H[v] = N_{coh}(v) := \max k : \exists j \text{ such that } f_j(v) = f_{j+1}(v) = \cdots = f_{j+k-1}(v)$$

Here operators $N_{coh}(v)$, for the fixed value v, shows the size of the groups that consist of the neurons that oscillate with the same spiking frequency f_j, ($j \in [1, N]$).

b) Clusters of subthreshold oscillations: In the case of a subthreshold oscillation cluster moving, $H[v]$ it can be equal to the size of the largest group of the neurons that oscillate with subthreshold frequency:

$$H[v] = N_{inh}(v) := |\{j : f_j(v) < thresh\}|$$

Here $N_{inh}(v)$ shows the maximum numbers of neurons demonstrating subthreshold oscillations.

c) Weighted sum: For our system with inhibitory synapses, we observed a complex multchimera where incoherent clusters consist of both asynchronous elements and elements that demonstrate subthreshold oscillations. So, we can choose the weighted sum of these heuristics:

$$H[v] = \frac{N_{coh}[v]}{||N_{coh}[v]||} + \frac{N_{inh}[v]}{||N_{inh}[v]||}, \quad (5)$$

Here, the norm is the measure of the spread of the given heuristic function: the maximum value minus the minimum value presented in the considered interval of speeds.

d) Frequency variance: We can develop another kind of heuristic. Consider a pair of frequency distribution graphs in Fig. 1. Let's go to a coordinate system where the chimera is static. The frequency distribution will have a wider spread compared to the original coordinates. Then we are free to take the frequency variance to measure the correctness of the choice of the chimera speed.

$$H[v] = D[v] = \sum_{j=1}^{N} (f_j[v] - \langle f_i[v] \rangle_i)^2 \quad (6)$$

where $f_i[v] = f[V((i + tv) \bmod N, t)]$ and $\langle f_i[v] \rangle_i$ is the spatial average (simulations not shown).

Then, at different values of the speed, we will move to the travelling coordinate system and calculate the frequency series. The speed at which this series will reach its maximum will be our desired chimera speed.

IV. TEST RESULTS

We tested different types of heuristic functional $H[v]$ and Fourier spectrum method by calculating the speed for a test set of travelling chimeras (see Suppl.). The test results are shown in the Table II. For the Fourier spectrum method, we calculated a dynamic that lasted 10 seconds. For the new method, we calculated a dynamic of 2 seconds.

We can see examples of how different heuristics work better or worse depending on the chimera's complexity and characteristics. In fact, the $N_{coh}[v]$ heuristic works with excellent accuracy for all the tests except when only clusters of subthreshold oscillations are observed. Adding $N_{inh}[v]$, we increase the accuracy of the speed calculation, and also expand the capabilities of the method. The variance-based heuristic and the Fourier method work well only when the chimera is well developed. This means that a big frequency spread is required for their correct work (see tests 4, 7, 8, 9).

Thus, the new method has several advantages over the previous one:

1) It does not depend on the number of incoherent clusters in a multichimera. This is important because there are no robust methods for such calculations.
2) Its accuracy does not depends on the length of the time series that is fed to it as input.
3) It works well for very low chimera speeds, making it possible to track a parametrically-driven smooth stop of the chimera.
4) It works well even for a complex types of chimera, including domains of subthreshold oscillations, as well as phase clustering.
5) It can be implemented as an automatic function.
6) It is not tied to periodic network topology

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SUPPLEMENTARY

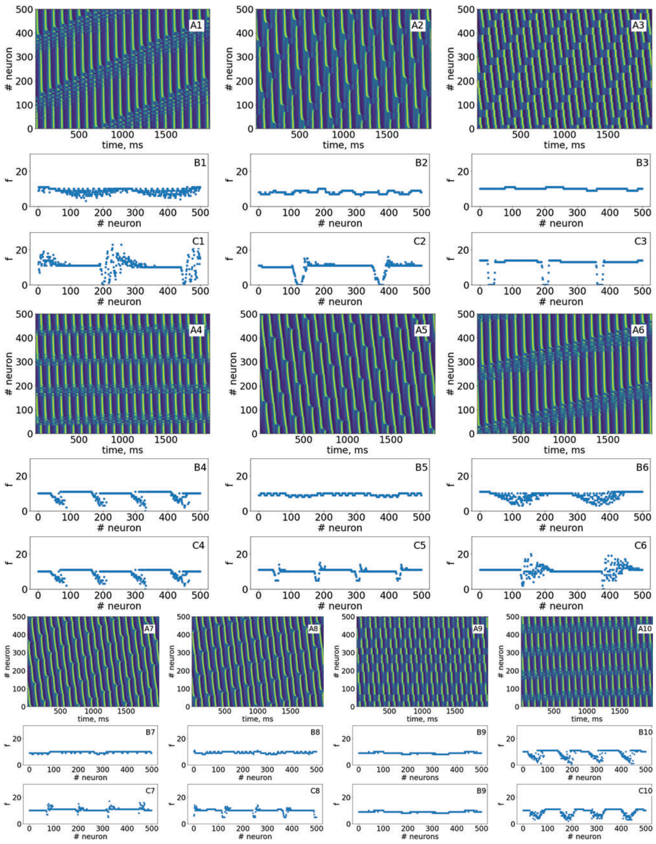

Figure 2. Test set. Rasterplots (A) in the case of travelling chimera state and corresponding frequency diagrams in initial (B) and traveling wave coordinates (C).