Anisotropic Superconductors Between Types I and II

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EBS - São Carlos - Feb, 2019

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Anisotropic superconductors between types I and II

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Phys. Rev. B **99**, 024515 – Published 28 January 2019

Thanks to my co-authors!!

Contents

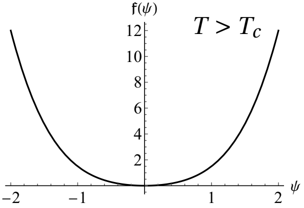
Crato-CE/Recife-PE

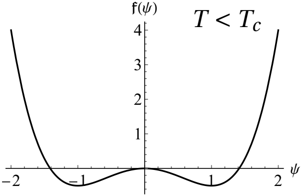
1 Ginzburg-Landau Theory

Free Energy functional in terms of the order parameter, Ψ, and vector potential, $\vec{A}$:

$$\mathfrak{F}[\Psi,\vec{A}] = \int dV \left[a\tau|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \mathcal{K}\left|\left(\vec{\nabla} + i\frac{Q}{\hbar c}\vec{A}\right)\Psi\right|^2 + \frac{(\vec{\nabla}\times\vec{A})^2}{8\pi}\right] \tag{1}$$

where $\tau = 1 - T/T_c$. Spontaneous breakdown of the symmetry at T_c



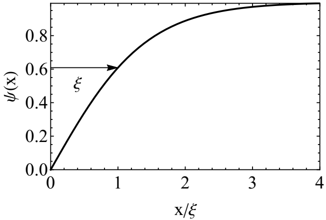


Minimization leads to the Ginzburg-Landau equations (with $\vec{D} = \vec{\nabla} + i\frac{Q}{\hbar c}\vec{A}$):

$$a\tau\Psi + b|\Psi|^2\Psi - \mathcal{K}\vec{D}^2\Psi = 0 \qquad (2)$$

$$\Psi_\infty = \sqrt{-a\tau/b} \Rightarrow \psi - \psi^3 + \psi'' = 0$$

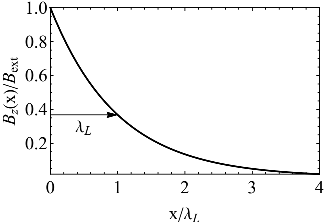
$$\Psi = \Psi_\infty \tanh(x/\xi\sqrt{2}), \qquad \xi^2 = \frac{\mathcal{K}}{|a|}$$



$$\frac{1}{4\pi}\vec{\nabla}\times\vec{B} = i\mathcal{K}\frac{Q}{\hbar c}\left[\Psi^*\vec{D}\Psi - \Psi\vec{D}^*\Psi^*\right] \quad (3)$$

$$\nabla^2\vec{B} = \frac{4\pi\mathcal{K}Q^2\Psi_\infty^2}{\hbar^2c^2}\vec{B}$$

$$B_z(x) = B_{ext}\exp(-x/\lambda_L), \qquad \lambda_L^2 = \frac{\hbar^2c^2}{4\pi\mathcal{K}Q^2\Psi_\infty^2}$$

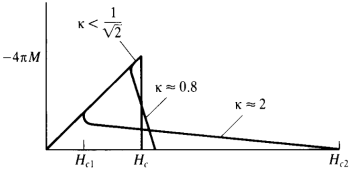


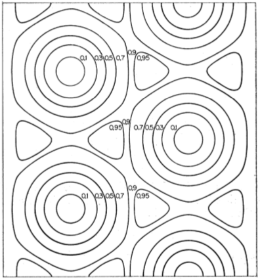
1.1 Abrikosov Classification of Superconducting Types

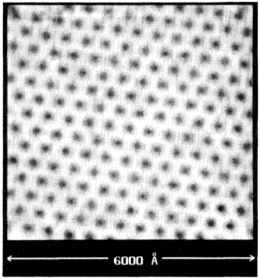
Abrikosov's solution to the linearized GL equation controlled by the Ginzburg-Landau parameter, $\kappa = \lambda/\xi$. Type-I × Type-II superconductors: the vortex phase.

$$-\left(\vec{\nabla} + i\frac{Q}{\hbar c}\vec{A}\right)^2 \Psi = \frac{1}{\xi^2}\Psi, \quad \vec{A} = Hx\hat{j}$$

$$H_{c2} = \sqrt{2}\kappa H_c$$

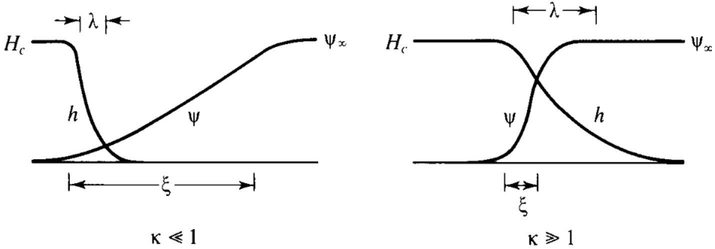






$$\mathfrak{G}_s = \int \mathfrak{g}_s d^3x, \qquad \mathfrak{g}_s = \mathfrak{f}_s + \frac{H_c^2}{8\pi} - \frac{H_c B}{4\pi}, \tag{4}$$

$$\sigma_{sn} = \int_{-\infty}^{\infty} dx \left[-\frac{|\Psi|^4}{2} + \frac{\left(\vec{B} - \vec{H}_c\right)^2}{8\pi} \right]$$



2 Bogomolny Equations (2D systems)

Define the operators

$$\Pi_{\pm} = D_x \pm i D_y \tag{5}$$

One can easily prove that

$$\Pi_{+}\Pi_{-} = \vec{D}^2 + |B_z|. \tag{6}$$

Then, when $\Pi_{-}\psi = 0$, one has

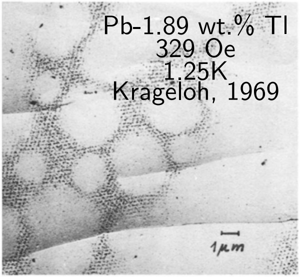
$$\psi - |\psi|^2\psi + \vec{D}^2\psi = 0$$
$$\psi - |\psi|^2\psi - |B_z|\psi = 0$$

$$|B_z| = 1 - |\psi|^2$$

$$\frac{1}{4\pi}\vec{\nabla} \times \vec{B} = i\mathcal{K}\frac{Q}{\hbar c}\left[\Psi^*\vec{D}\Psi - \Psi\vec{D}^*\Psi^*\right] \tag{7}$$

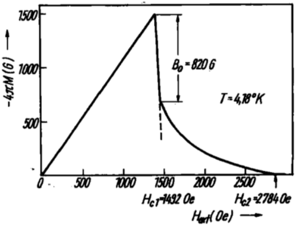
$$\kappa = \kappa_0 = 1/\sqrt{2}$$

Anomalous behavior close to the Bogomolny point

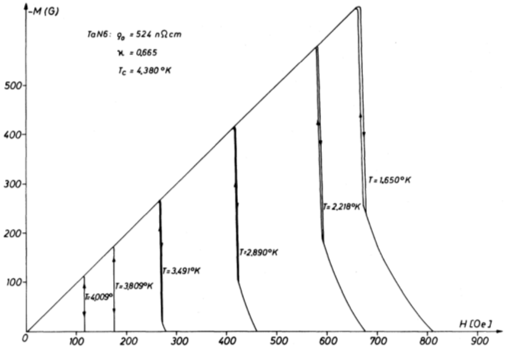


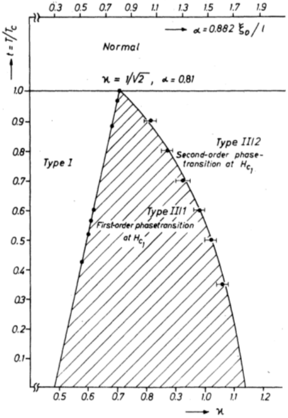
Pb-1.89 wt.% Tl
329 Oe
1.25K
Krageloh, 1969

Nb s.c. 4.18K Kumpf, 1971



Auer and Ullmaier, (1973): TaN ($\kappa = 0.665$: Type-I in principle)





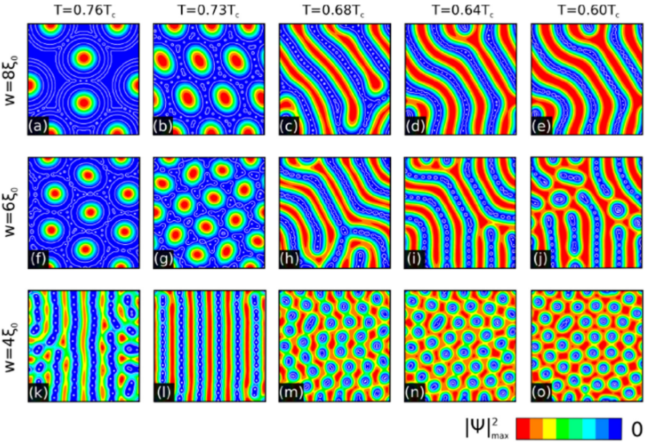


FIG. 2. The local density of Cooper pairs $|\Psi|^2$ for film thicknesses $w/\xi_0 = 8$ panels (a)–(e), $w/\xi_0 = 6$ [panels (f)–(j)], and $w/\xi_0 = 4$ [panels (k)–(o)], calculated at temperatures $T/T_c = 0.76, 0.73, 0.68, 0.64, 0.6$. Other parameters are the same as in Fig. 1.

PRB 94, 054511 (2016)

2.1 Theoretical Problem

Type-1.5 superconductivity in two-band systems

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PHYSICAL REVIEW B **83**, 054515 (2011)

Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity

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(Received 2 August 2010; published 23 February 2011)

PHYSICAL REVIEW B **86**, 016501 (2012)

Comment on “Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity”

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(Received 18 May 2011; revised manuscript received 22 February 2012; published 5 July 2012)

PHYSICAL REVIEW B **86**, 016502 (2012)

Reply to “Comment on ‘Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity’”

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3 Extended Ginzburg-Landau Theory

BCS Hamiltonian for $N = 1,\ 2,\ 3...$ overlapping bands (Suhl et al. PRL 1959):

$$\mathcal{H}_{\text{BCS}} = \sum_{i=1}^{N} \left\{ \sum_{\sigma=\uparrow,\downarrow} \int d^D x \psi^\dagger_{i\sigma}(\vec{x}) \mathcal{T}_{ix} \psi_{i\sigma}(\vec{x}) + \int d^D x \left[\psi^\dagger_{i\uparrow}(\vec{x}) \psi^\dagger_{i\downarrow}(\vec{x}) \Delta_i(\vec{x}) + H.c. \right] \right\} \tag{8}$$

where $\mathcal{T}_{ix}$ is the single-electron kinetic energy operator of nth band and the energy gap are

$$\mathcal{T}_{ix} \equiv -\frac{\hbar^2}{2m_e} \left(\vec{\nabla} - i \frac{e}{\hbar c} \vec{A} \right)^2 - \mu_i \qquad \Delta_i(\vec{x}) = \sum_{j=1}^{N} g_{ij} \langle \psi_{j\uparrow}(\vec{x}) \psi_{j\downarrow}(\vec{x}) \rangle . \tag{9}$$

Complex time formalism: $\mathfrak{t} = it$ and operators in the Heisenberg picture

$$\psi_{i\sigma}(\vec{x},\mathfrak{t}) = \exp(\mathcal{H}_{\mathrm{BCS}}\mathfrak{t}/\hbar)\ \psi_{i\sigma}(\vec{x})\ \exp(-\mathcal{H}_{\mathrm{BCS}}\mathfrak{t}/\hbar), \tag{10}$$

$$\bar{\psi}_{i\sigma}(\vec{x},\mathfrak{t}) = \exp(\mathcal{H}_{\mathrm{BCS}}\mathfrak{t}/\hbar)\ \psi^{\dagger}_{i\sigma}(\vec{x})\ \exp(-\mathcal{H}_{\mathrm{BCS}}\mathfrak{t}/\hbar). \tag{11}$$

$$-\hbar\partial_{\mathfrak{t}}\psi_{i\uparrow}(\vec{x},\mathfrak{t}) = \left[\psi_{i\uparrow}(\vec{x},\mathfrak{t}),\mathcal{H}_{\mathrm{BCS}}\right] = \mathcal{T}_x\psi_{i\uparrow}(\vec{x},\mathfrak{t}) + \Delta(\vec{x})\bar{\psi}_{i\uparrow}(\vec{x},\mathfrak{t}) \tag{12}$$

$$-\hbar\partial_{\mathfrak{t}}\bar{\psi}_{i\downarrow}(\vec{x},\mathfrak{t}) = \left[\bar{\psi}_{i\downarrow}(\vec{x},\mathfrak{t}),\mathcal{H}_{\mathrm{BCS}}\right] = -\mathcal{T}^{*}_x\bar{\psi}_{i\downarrow}(\vec{x},\mathfrak{t}) + \Delta^{*}(\vec{x})\psi_{i\uparrow}(\vec{x},\mathfrak{t}) \tag{13}$$

SOVIET PHYSICS JETP VOLUME 36(9), NUMBER 6 DECEMBER, 1959

MICROSCOPIC DERIVATION OF THE GINZBURG-LANDAU EQUATIONS IN THE THEORY OF SUPERCONDUCTIVITY

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Submitted to JETP editor February 3, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 36, 1918-1923 (June, 1959)

$$
\begin{aligned}
-\hbar\partial_{\mathrm{t}}\mathcal{G}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') &= \partial_{\mathrm{t}}\langle T_{\mathrm{t}}\psi_{i\uparrow}(\vec{x},\mathrm{t})\bar{\psi}_{i\uparrow}(\vec{x},\mathrm{t})\rangle \\
&= \delta(\mathrm{t}-\mathrm{t}')\delta(\vec{x}-\vec{x}') + \mathcal{T}_{ix}\mathcal{G}_i(\vec{x}\mathrm{t};\vec{x}'\mathrm{t}') + \Delta_i(\vec{x})\bar{\mathcal{F}}_i(\vec{x}\mathrm{t};\vec{x}'\mathrm{t}'), \quad (14)\\
-\hbar\partial_{\mathrm{t}}\bar{\mathcal{F}}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') &= \partial_{\mathrm{t}}\langle T_{\mathrm{t}}\bar{\psi}_{i\downarrow}(\vec{x},\mathrm{t})\bar{\psi}_{i\uparrow}(\vec{x},\mathrm{t})\rangle \\
&= \Delta_i^*(\vec{x})\mathcal{G}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') - \mathcal{T}_{ix}^*\bar{\mathcal{F}}_i(\vec{x},\mathrm{t};\vec{x}'\mathrm{t}'), \quad (15)\\
-\hbar\partial_{\mathrm{t}}\bar{\mathcal{G}}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') &= \partial_{\mathrm{t}}\langle T_{\mathrm{t}}\bar{\psi}_{i\downarrow}(\vec{x},\mathrm{t})\psi_{i\downarrow}(\vec{x},\mathrm{t})\rangle \\
&= \delta(\mathrm{t}-\mathrm{t}')\delta(\vec{x}-\vec{x}') + \Delta_i^*(\vec{x})\mathcal{F}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') - \mathcal{T}_{ix}^*\bar{\mathcal{G}}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}'), \quad (16)\\
-\hbar\partial_{\mathrm{t}}\mathcal{F}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') &= \partial_{\mathrm{t}}\langle T_{\mathrm{t}}\psi_{i\uparrow}(\vec{x},\mathrm{t})\psi_{i\downarrow}(\vec{x},\mathrm{t})\rangle \\
&= \mathcal{T}_{ix}^*\mathcal{F}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') - \Delta_i(\vec{x})\bar{\mathcal{F}}_i(\vec{x},\mathrm{t};\vec{x}',\mathrm{t}') \quad (17)
\end{aligned}
$$

where T_{t} is the time-ordering operator.

The Fourier components of the Green functions $\mathcal{Y} = \{\mathcal{G}, \bar{\mathcal{G}}, \mathcal{F}, \bar{\mathcal{F}}\}$ are given by

$$\mathcal{Y}(\vec{x}, \mathfrak{t}; \vec{x}', \mathfrak{t}') = \frac{1}{\beta\hbar} \sum_{n=-\infty}^{\infty} \exp\left[-i\omega_n(\mathfrak{t}' - \mathfrak{t})\right] \mathcal{Y}_{\omega_n}(\vec{x}, \vec{x}'), \tag{18}$$

$$\mathcal{Y}_{\omega_n}(\vec{x}, \vec{x}') = \frac{1}{2} \int_{-\hbar\beta}^{\hbar\beta} \mathrm{d}\eta \; \exp\left[i\omega_n(\mathfrak{t}' - \mathfrak{t})\right] \mathcal{Y}(\vec{x}, \mathfrak{t}; \vec{x}', \mathfrak{t}) \tag{19}$$

where $\omega_n = \pi(2n+1)/\beta\hbar$ are the fermionic Matsubara frequencies. In the absence of condensate, one has Unperturbed Green's functions:

$$\mathcal{G}^{(0)}_{i\omega}(\vec{z} = \vec{x}' - \vec{x}) = -\frac{\pi N_i(0)}{k_i z} \exp\left[-i\frac{e}{\hbar c}\int_{\vec{x}'}^{\vec{x}} \vec{A}(\vec{y}) \cdot d\vec{y}\right] \exp\left[i\mathrm{sgn}(\omega) k_i z - \frac{|\omega|}{v_i} z\right] \tag{20}$$

where $N_i(0) = \frac{m_e k_i}{2\pi^2\hbar^2}$ is the density of states at the Fermi surface. $\Rightarrow \xi_{i0} \sim \frac{\hbar v_i}{\pi T_c}$

$$
\begin{aligned}
\mathfrak{F}_s = \mathfrak{F}_n + \int \mathrm{d}^3 x \Bigg[\frac{\vec{B}^2(\vec{x})}{8\pi} + \vec{\Delta}\cdot\gamma\cdot\vec{\Delta} - \sum_{i=1}^{N} \int \mathrm{d}^3 x K_{ia}(\vec{x},\vec{y})\Delta_i^*(\vec{x})\Delta_i(\vec{y}) \\
- \frac{1}{2}\int \left(\prod_{j=1}^{3} \mathrm{d}^3 y_j\right) K_{ib}(\vec{x},\vec{y}_1,\vec{y}_2,\vec{y}_3)\Delta_i^*(\vec{x})\Delta_i(\vec{y}_1)\Delta_i^*(\vec{y}_2)\Delta_i(\vec{y}_3) \\
- \frac{1}{3}\int \left(\prod_{j=1}^{5} \mathrm{d}^3 y_j\right) K_{ic}(\vec{x},\vec{y}_1,\vec{y}_2,\vec{y}_3,\vec{y}_4,\vec{y}_5)\Delta_i^*(\vec{x})\Delta_i(\vec{y}_1)\Delta_i^*(\vec{y}_2)\Delta_i(\vec{y}_3)\Delta_i^*(\vec{y}_4)\Delta_i(\vec{y}_5) - \dots \Bigg],
\end{aligned}
$$

$$K_{ia}(\vec{x},\vec{y}) = -gT \lim_{\mathfrak{t}'-\mathfrak{t}\to 0^+} \sum_{\omega} \exp[-i\omega(\mathfrak{t}'-\mathfrak{t})]\mathcal{G}_{i\omega}^{(0)}(\vec{x},\vec{y})\bar{\mathcal{G}}_{i\omega}^{(0)}(\vec{y},\vec{x}), \tag{21}$$

$$K_{ib}(\vec{x},\vec{y}_1,\vec{y}_2,\vec{y}_3) = -gT\sum_{\omega}\mathcal{G}_{i\omega}^{(0)}(\vec{x},\vec{y}_1)\bar{\mathcal{G}}_{i\omega}^{(0)}(\vec{y}_1,\vec{y}_2)\mathcal{G}_{i\omega}^{(0)}(\vec{y}_2,\vec{y}_3)\bar{\mathcal{G}}_{i\omega}^{(0)}(\vec{y}_3,\vec{x}), \tag{22}$$

$$K_{ic}(\vec{x},\vec{y}_1,...,\vec{y}_5) = -gT\sum_{\omega}\mathcal{G}_{i\omega}^{(0)}(\vec{x},\vec{y}_1)\bar{\mathcal{G}}_{i\omega}^{(0)}(\vec{y}_1,\vec{y}_2)\ \dots\ \bar{\mathcal{G}}_{i\omega}^{(0)}(\vec{y}_3,\vec{y}_4)\mathcal{G}_{i\omega}^{(0)}(\vec{y}_4,\vec{y}_5)\bar{\mathcal{G}}_{i\omega}^{(0)}(\vec{y}_5,\vec{x}). \tag{23}$$

Series expansion of the gaps inside integrals due to divergence of typical lengths close to T_c. Defining $\vec{z} = \vec{y} - \vec{x}$, then

$$\Delta_i(\vec{y}) = \sum_{j=0}^{\infty} \frac{(\vec{\nabla} \cdot \vec{z})^j}{j!} \Delta_i(\vec{x}) \tag{24}$$

Expansion of the relevant quantities in powers of τ

$$\vec{\Delta}(\vec{x}) = \tau^{1/2} \left[\vec{\Delta}^{(0)}(\vec{x}) + \tau \vec{\Delta}^{(1)}(\vec{x}) + \mathcal{O}(\tau^2) \right] \tag{25}$$

$$\vec{A}(\vec{x}) = \tau^{1/2} \left[\mathfrak{A}^{(0)}(\vec{x}) + \tau \mathfrak{a}^{(1)}(\vec{x}) + \mathcal{O}(\tau^2) \right] \tag{26}$$

$$\vec{B}(\vec{x}) = \tau^{1/2} \left[\mathfrak{B}^{(0)}(\vec{x}) + \tau \mathfrak{b}^{(1)}(\vec{x}) + \mathcal{O}(\tau^2) \right] \tag{27}$$

Standard perturbation theory: collection terms of equal power in τ, in special

$$\vec{\mathfrak{D}} \Delta_i^{(n)}(\vec{x}) = \left(\vec{\nabla} - i \frac{2e}{\hbar c} \vec{\mathfrak{A}} \right) \Delta_i^{(n)}(\vec{x}) \propto \mathcal{O}(\tau^{n+1}). \tag{28}$$

PRL **106,** 047005 (2011) PHYSICAL REVIEW LETTERS week ending 28 JANUARY 2011

Extended Ginzburg-Landau Formalism for Two-Band Superconductors

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(Received 18 October 2010; published 27 January 2011)

$$\mathfrak{f}_s = \tau^2 \left(\tau^{-1}\mathfrak{f}^{(-1)} + \mathfrak{f}^{(0)} + \tau\mathfrak{f}^{(1)} + \ldots\right),$$

$$\mathfrak{f}^{(-1)} = \vec{\Delta}^{(0)} \cdot L \cdot \vec{\Delta}^{(0)}, \tag{29}$$

$$\mathfrak{f}^{(0)} = \frac{\vec{\mathfrak{B}}^2}{8\pi} + \left(\vec{\Delta}^{(0)} \cdot L \cdot \vec{\Delta}^{(1)} + c.c.\right) + \sum_i \left[a_i|\Delta_i^{(0)}|^2 + \frac{b_i}{2}|\Delta_i^{(0)}|^4 + \mathcal{K}_i|\mathfrak{D}\Delta_i^{(0)}|^2\right], \tag{30}$$

$$\mathfrak{f}^{(1)} = \frac{\vec{\mathfrak{B}} \cdot \vec{\mathfrak{b}}}{2\pi} + \left(\vec{\Delta}^{(0)} \cdot L \cdot \vec{\Delta}^{(2)} + c.c.\right) + \vec{\Delta}^{(1)} \cdot L \cdot \vec{\Delta}^{(1)} + \sum_i \left(\mathfrak{f}^{(1)}_{i,1} + \mathfrak{f}^{(1)}_{i,2}\right), \tag{31}$$

$$\begin{aligned}\mathfrak{f}^{(1)}_{i,1} = {} & \frac{a_i}{2}|\Delta_i^{(0)}|^2 + 2\mathcal{K}_i|\vec{\mathfrak{D}}\Delta_i^{(0)}|^2 + \frac{b_i}{36}\frac{e^2\hbar^2}{m^2c^2}\vec{\mathfrak{B}}^2|\Delta_i^{(0)}|^2 + b_i|\Delta_i^{(0)}|^4 \\ & - \mathcal{Q}_i\left\{|\vec{\mathfrak{D}}^2\Delta_i^{(0)}|^2 + \frac{1}{3}(\vec{j}_i \cdot \vec{\nabla} \times \vec{\mathfrak{B}}) + \frac{4e^2}{\hbar^2c^2}\vec{\mathfrak{B}}^2|\Delta_i^{(0)}|^2\right\} \\ & - \frac{\mathcal{L}_i}{2}\left\{8|\Delta_i^{(0)}|^2|\mathfrak{D}\Delta_i^{(0)}|^2 + \left[\Delta_i^{(0)2}\left(\mathfrak{D}^*\Delta_i^{*(0)}\right)^2 + c.c.\right]\right\},\end{aligned} \tag{32}$$

$$\mathfrak{f}^{(1)}_{i,2} = \left(a_i + b_i|\Delta_i^{(0)}|^2\right)\left(\Delta_i^{*(0)}\Delta_i^{(1)} + c.c.\right) + \mathcal{K}_i\left[\left(\vec{\mathfrak{D}}\Delta_i^{(0)} \cdot \vec{\mathfrak{D}}^*\Delta_i^{*(1)} + c.c.\right) - (\vec{\mathfrak{a}} \cdot \vec{j}_i)\right] \tag{33}$$

Gor'kov			Shanenko et al.		
a_i	b_i	$\mathcal{K}_i$	c_i	$\mathcal{Q}_i$	$\mathcal{L}_i$
$-N_i(0)$	$\frac{7\zeta(3)}{8\pi^2 T_c^2} N_i(0)$	$\frac{b_i}{6}\hbar^2 v_i^2$	$\frac{98\zeta(5)}{128\pi^4 T_c^4} N_i(0)$	$\frac{c_i}{30}\hbar^4 v_i^4$	$\frac{c_i}{9}\hbar^2 v_i^2$

where

$N_i(0)$: Density of states at the Fermi level for band $i = 1, ..., N$

v_i: Fermi velocity for band $i = 1, ..., N$

Expressing relevant quantities in dimensionless units:

$$\vec{x} \to \lambda_L \sqrt{2}\vec{x}, \quad \vec{\mathfrak{A}} \to \frac{H_c^{(0)} \lambda_L}{\kappa} \vec{\mathfrak{A}}, \quad \vec{\mathfrak{B}} \to \frac{H_c^{(0)}}{\sqrt{2}\kappa} \vec{\mathfrak{B}}, \quad \Delta^{(0)} \to \Psi_\infty \Psi \tag{34}$$

the Gibbs free energy difference becomes

$$\mathfrak{g}_s = \tau^2 \left(\mathfrak{g}_s^{(0)} + \tau \mathfrak{g}_s^{(1)} + \ldots \right), \tag{35}$$

$$\mathfrak{g}_s^{(0)} = \frac{1}{2} \left(\frac{|\vec{\mathfrak{B}}|}{\sqrt{2}\kappa} - 1 \right)^2 + \frac{1}{\sqrt{2}\kappa^2} |\vec{\mathfrak{D}}\Psi|^2 - |\Psi|^2 + \frac{1}{4}|\Psi|^4, \tag{36}$$

$$\begin{aligned} \mathfrak{g}_s^{(1)} = & \left(\frac{|\vec{\mathfrak{B}}|}{\sqrt{2}\kappa} - 1 \right) \left(\frac{1}{2} + \frac{ac}{3b^2} \right) - \frac{|\Psi|^2}{2} + |\Psi|^4 + \frac{|\vec{\mathfrak{D}}\Psi|^2}{\kappa^2} + \frac{1}{4\kappa^4} \frac{a\mathcal{Q}}{\mathcal{K}^2} \left\{ |\vec{\mathfrak{D}}^2 \Psi|^2 + \frac{1}{3} (\vec{\nabla} \times \vec{\mathfrak{B}})^2 + \vec{\mathfrak{B}}^2 |\Psi|^2 \right\} \\ & + \frac{1}{4\kappa^2} \frac{a\mathcal{L}}{b\mathcal{K}} \left\{ 8|\Psi|^2 |\vec{\mathfrak{D}}\Psi|^2 + [\Psi^2 (\vec{\mathfrak{D}}^* \Psi^*)^2 + c.c] \right\} + \frac{ac}{3b^2} |\Psi|^6. \end{aligned} \tag{37}$$

With this expression, it is possible to determine the boundaries between types I and II in the (κ, T) plane by expanding $\mathfrak{G}_s$ around $\kappa_0 = 1/\sqrt{2}$

$$\mathfrak{G}_s = \tau^2 \left(\mathfrak{G}_s^{(0)} + \frac{d\mathfrak{G}_s^{(0)}}{d\kappa}\bigg|_{\kappa=\kappa_0} \delta\kappa + \tau \mathfrak{G}^{(1)} + \dots \right) \tag{38}$$

where $\delta\kappa = \kappa - \kappa_0$. Simplification from Bogomolnyi self-dual equations:

$$\mathfrak{G}_s = \tau^2 \left\{ -\sqrt{2}\mathcal{I}\delta\kappa + \tau \left[\left(1 - \frac{ac}{3b^2} + 2\frac{a\mathcal{Q}}{\mathcal{K}^2} \right) \mathcal{I} + \left(2\frac{a\mathcal{L}}{b\mathcal{K}} - \frac{ac}{3b^2} - \frac{5}{3}\frac{a\mathcal{Q}}{\mathcal{K}^2} \right) \mathcal{J} \right] + \dots \right\} \tag{39}$$

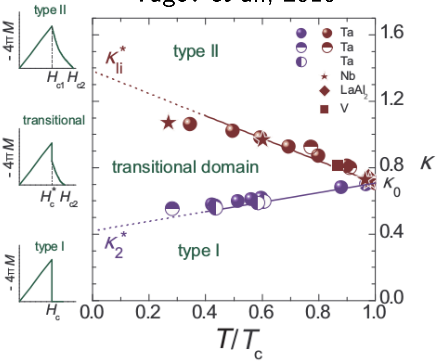
where $\mathcal{I} = \int |\Psi|^2(1 - |\Psi|^2) d^3x$ and $\mathcal{J} = \int |\Psi|^4(1 - |\Psi|^2) d^3x$.

Nucleation condition, $\mathfrak{G}_s(\kappa, T) = 0$, provides

$$\kappa^* = \kappa_0 \left\{ 1 + \tau \left[1 - \frac{ac}{3b^2} + 2\frac{a\mathcal{Q}}{\mathcal{K}^2} + \frac{\mathcal{J}}{\mathcal{I}} \left(2\frac{a\mathcal{L}}{b\mathcal{K}} - \frac{ac}{b^2} - \frac{5}{3}\frac{a\mathcal{Q}}{\mathcal{K}^2} \right) \right] \right\}$$

Configuration of Ψ where the onset of attractive interaction between two vortices: $\mathcal{J}/\mathcal{I} \to 2$
Configuration of Ψ for equal thermodynamic critical fields $\mathcal{J}/\mathcal{I} \to 0$.

Vagov et al., 2016



4 Anisotropy and the Inter-Type Domain

Consider an anisotropic kinetic part of the Hamiltonian

$$\mathcal{T} = -\sum_{j=1}^{3} \frac{\hbar^2}{2m_j} \left(\partial_j - i\frac{e}{\hbar c} A_j\right)^2 - \mu. \tag{40}$$

The domain of integration of the unperturbed Green function is the elliptic Fermi surface and the dispersion relation becomes anisotropic

$$\mathcal{G}_\omega^{(0)}(\vec{x},\vec{x}') = \exp\left[-i\frac{e}{\hbar c}\int_{\vec{x}'}^{\vec{x}} \vec{A}(\vec{y})\cdot d\vec{y}\right]\int \frac{d^3k}{(2\pi)^3}\frac{\exp[i\vec{k}\cdot(\vec{x}-\vec{x}')]}{i\hbar\omega - \xi_k}, \qquad \xi_k = \sum_{j=1}^{3}\frac{\hbar^2}{2m_j}k_j^2 - \mu. \tag{41}$$

By a matter of scaling, one can map this Hamiltonian into the isotropic case.

$$\tilde{x}_i = \frac{1}{\sqrt{\alpha_i}} x_i, \quad \tilde{A}_i = \sqrt{\alpha_i} A_i, \quad \tilde{B}_i = \frac{1}{\sqrt{\alpha_i}} B_i, \tag{42}$$

This scaling must result in a unique electronic mass M for each direction

$$-\sum_{j=1}^{3} \frac{\hbar^2}{2m_j} \left(\partial_j - i\frac{e}{\hbar c} A_j\right)^2 - \mu \qquad \rightarrow \qquad -\sum_{i=1}^{3} \frac{\hbar^2}{2M} \left(\tilde{\partial}_j - i\frac{e}{\hbar c} \tilde{A}_i\right)^2 - \mu \tag{43}$$

and does not induce alteration in the elements of volume after this variable change

$$\left.\begin{array}{l} \alpha_i m_i = \alpha_j m_j = M \quad (\forall \ i, j), \\ \prod_{i=1}^{3} \sqrt{\alpha_i} = 1 \end{array}\right\} \Rightarrow \alpha_i = \frac{M}{m_i}, \quad M = \sqrt[3]{m_x m_y m_z}. \tag{44}$$

This mapping works for arbitrary order of expansion of the gap! In particular, 1st order

$$\begin{array}{l} a\Delta_0 + b|\Delta_0|^2\Delta_0 - \tilde{\mathcal{K}}\tilde{\mathcal{D}}^2\Delta_0 = 0 \\ \tilde{\nabla}\times\tilde{B}_0 = 4\pi\tilde{\mathcal{K}}i\frac{2e}{\hbar c}\left(\Delta_0\tilde{\mathcal{D}}^*\Delta_0^* - \Delta_0^*\tilde{\mathcal{D}}\Delta_0\right) \end{array} \rightarrow \left\{ \begin{array}{l} a\Delta^{(0)} + b|\Delta^{(0)}|^2\Delta^{(0)} - \left(\mathcal{K}_x\mathcal{D}_x^2 + \mathcal{K}_y\mathcal{D}_y^2\right)\Delta^{(0)} = 0 \\ \frac{1}{\alpha_z}\partial_y B = 4\pi\mathcal{K}_x i\frac{2e}{\hbar c}\left(\Delta_0\mathcal{D}_x^*\Delta_0^* - \Delta_0^*\mathcal{D}_x\Delta_0\right) \\ \frac{1}{\alpha_z}\partial_x B = -4\pi\mathcal{K}_y i\frac{2e}{\hbar c}\left(\Delta_0\mathcal{D}_y^*\Delta_0^* - \Delta_0^*\mathcal{D}_y\Delta_0\right). \end{array}\right. \tag{45}$$

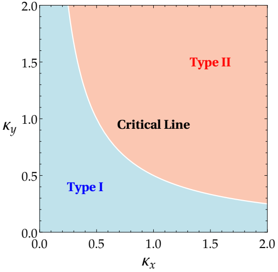
Renormalization of DOS': $a = -\alpha_z N(0)$, $b = \alpha_z N(0)\frac{7\zeta(3)}{8\pi^2 T_c^2}$ and $\mathcal{K}_j = \frac{b}{2}\hbar^2 v_j^2$. From this set of equations one extracts the characteristic lengths

$$\xi_j^{(z)} = \sqrt{\frac{\mathcal{K}_j}{|a|}}, \quad \lambda_j^{(z)} = \sqrt{\frac{\hbar^2 c^2 b,}{32\pi^2 e^2\mathcal{K}_j|a|}} \quad \Rightarrow \kappa^{(z)} = \sqrt{\kappa_x^{(z)}\kappa_y^{(z)}} \tag{46}$$

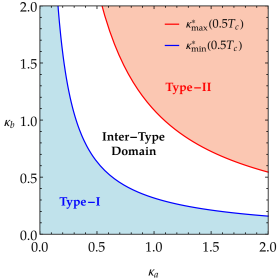
where $j = x,\ y$ and the superscript z reflects the field direction.

Intertype domain in the $\kappa_x \times \kappa_y$ diagram and comparison with experimental results from Weber, (1978)

$$\sqrt{\kappa_x \kappa_y} = \kappa_0$$



$$\sqrt{\kappa_x \kappa_y} = \kappa_0 + \delta\kappa + \mathcal{O}(\tau^2)$$



Transition from Type-II to Type-I Superconductivity with Magnetic Field Direction

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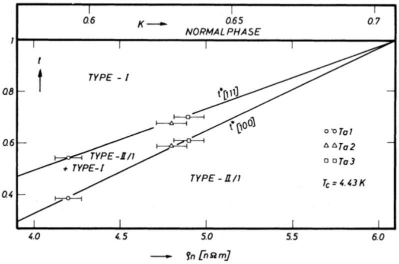
(Received 11 August 1978)

We report on a new effect in superconductivity and demonstrate experimentally that, because of the correlation of the upper critical field H_{c2} with crystal directions in single-crystalline *Ta*N samples (anisotropy effect), at certain fixed temperatures the material is a type-I superconductor near the [100] *and* a type-II superconductor near the [111] directions.



Obrigado!!

Thank you!!

Aknowledgements:

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

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